An Investigation into the Empty Universe

Abstract

The FLRW empty or Milne universe, where the relative energy density of curvature is set to one, is expanding at the speed of light. But what is expanding? The most obvious answer is a 3-sphere. But if the mass equivalent of the curvature energy exists in the 3-sphere 'shell', it will have gravitational potential energy even though there seems to be no room for this potential energy in the empty universe. But if, however, this potential energy, which is negative, is balanced by components with positive energy, such as matter and kinetic energy, the total energy of the components will be zero and they will be invisible to the FLRW metric. The 'empty' universe need not be empty.

This paper goes on to show that such an 'empty' or 3-sphere universe with components makes a number of extremely good predictions and should be given serious consideration as a contender model for the universe we live in.

Introduction

The empty universe, sometimes known as the Milne universe[1], is what results, when in the FLRW metric, the relative energy density of curvature is set to 1. Since all of the relative energy densities of the components in an FLRW metric must add to 1, it follows that all of the other components must have a relative energy density of zero. It is sometimes said that the Milne universe has an energy density of zero, but of course this is ignoring the energy density of curvature itself.

The empty or Milne universe is often included in diagrams showing the possible evolution of the universe but is not a serious contender as a model of our universe, presumably because it is empty. This paper discusses the possibility that the empty universe may be more interesting than generally acknowledged.

1. The FLRW metric.

The FLRW metric is the family of solutions to Einstein's field equations of GR when applied to a universe that is homogeneous, isotropic, can expand (or contract) and can contain components. It consists of two linked differential equations plus an energy balance. The third, the energy balance, will suffice here, Equation 1[2].

$$H_{(z)}/H_0 = \sqrt{(\Omega_r(1+z)^4 + (\Omega_c + \Omega_b)(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda)} \qquad \text{Equation 1}$$

z – the red shift associated with the universe at a particular age. $1+z = 1/a$, *a* being the scale factor

$H_{(z)}$ – The Hubble parameter associated with a particular value of the red shift or scale factor, z=0 indicating today's universe

Ω – The energy density, relative to the critical energy density of the component indicated by the subscript. The Ω terms must therefore sum to 1.

The subscripts

r – radiation, mostly photons and neutrinos
c – cold dark matter
b – baryonic matter
k – curvature

Λ – dark energy or the cosmological constant

The superscripts determine how each component scales as the universe expands.

In the standard cosmological model, each component is in the region of

r – negligible, often considered to be zero
c – 0.25
b – 0.05
k – 0
Λ – 0.7

In the empty universe, r=c=b=Λ=0. k must therefore be 1. A positive value of k implies negative or hyperbolic curvature. More of this later. The FLRW metric for the empty universe simplifies to

$H_{(z)}/H_0 = \sqrt{(\Omega_k(1+z)^2)}$, where $\Omega_k = 1$ Equation 2

$H_{(z)}/H_0 = 1+z$ Equation 3

When the universe was half its current size, z=1, and so $H_{(z)} = 2H_0$. The empty universe is expanding at a constant rate. Consequently the age of the universe is exactly equal to the reciprocal of today's Hubble parameter, which if we take to be 71 km/s per megaparsec, corresponds to an age of 13.8 billion years.

2. The size of the empty universe

The formula for the radius of curvature of FLRW metrics[3] is $C/(H\sqrt{(|\Omega_k|)})$ which in the empty universe is simply C/H or, for an H value of 71, 13.8 billion light years. We have already seen that the empty universe is expanding at a constant rate and now we see that this rate is the speed of light. The empty universe is a Hubble sphere. But we need to ask, what is actually expanding? The empty universe cannot be an expanding ball because a ball is not homogeneous, which all FLRW metrics must be. It seems most likely that it is an expanding 3-sphere.

3. The mass of the empty universe

Since the empty universe is finite, it will have a finite mass. This can be estimated in more than one way.

If we assume the density is the critical density, then if we know the volume, we also know the mass. For H_0 of 71, the radius is 13.8 billion ly and the critical density is 9.47×10^{-27}. If we take the volume to be that of a 3D ball, $4/3\ \pi r^3$, the volume observed by an external observer, the total mass is 8.83×10^{52} kg, though if we take the volume to be that of a 3-sphere, $2\pi^2 r^3$, the internal volume, the total mass is 4.16×10^{53} kg.

Another way to estimate the mass is to assume that the empty universe is a 3-sphere. Then a photon is effectively in orbit at the speed of light. If we assume the Newtonian formula for an orbit

$M = c^2 r/G$ Equation 4

The total mass is 1.73×10^{53} kg. But if we instead assume the Schwarzschild relationship

$M = c^2 r/2G$ Equation 5

The total mass is 8.7 x 10^{52}. Assuming the Schwarzschild relationship for a photon in orbit gives the same mass as assuming the critical density and the volume of a 3D ball. Note that if we use either Equation 4 or Equation 5, the mass of the empty universe is proportional to its radius or scale factor.

4. The distance-scale factor relationship in the empty universe

If the empty universe is a 3-sphere, then the scale factor, *a,* is equal to r/r_0 where r is the radius. In addition, the red shift is given by $1/a$ -1, though one could be forgiven for asking how red shift applies to the empty universe. The relationship to distance can be calculated in two different ways, one, analytical, based on the FLRW metric, and the other, geometrical, based on the 3-sphere. The analytical method is based on equation 6.[4]

$$d_c = d_H \int_0^z 1/E_{(z)}\, dz \qquad \text{Equation 6}$$

d_c - co-moving radial distance, the distance from us in today's universe.

d_H - the Hubble distance, equal to c/H_0 (which is r_0, the radius of the empty universe, 13.8 billion ly according to Section 2)

$E_{(z)}$ - $H_{(z)}/H_0$ also equal to $1/a$ or (z+1) in the empty universe (Equation 3).

The solution to Equation 6 is $d_c = r_0 \ln(z+1)$ or, maybe more usefully, $z = \exp(d_c/r_0) - 1$

The geometric solution is based on the idea of the path of light in an expanding 3-sphere. This solution requires at least one photon and one infinitesimal point mass, neither of which should exist in the empty universe. This will be addressed later.

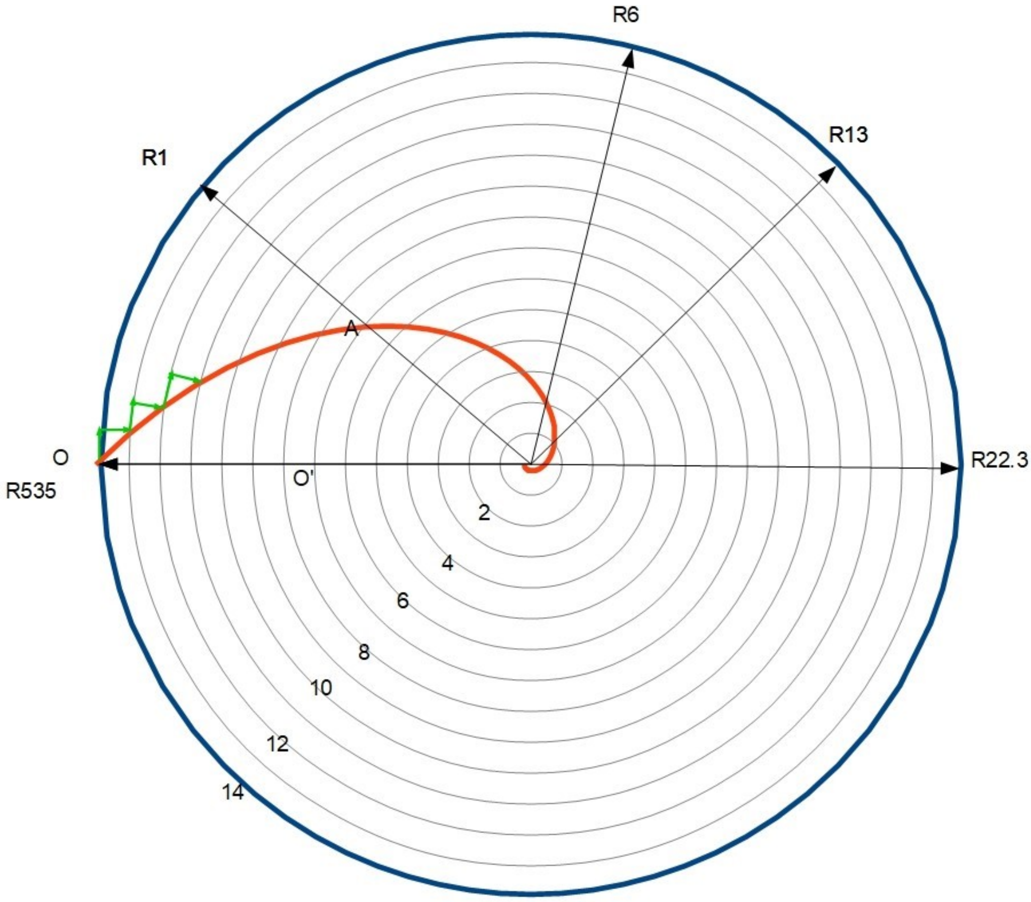

Figure 1. Red shift in an expanding 3-sphere universe.

For simplicity, this diagram assumes an age for the empty or 3-sphere universe of 14 billion years and a size of 14 billion light years. Each circle represents the entire 3-sphere universe at a particular age. The universe is expanding at the speed of light, see Section 2. We wish to find the path of a photon in this expanding universe but if we try to do this going forward in time, we don't know where to start. No matter, we can trace the path backwards in time, as it leaves an observer at 'O'. Let the photon move for a billion years in today's universe and then move with the contracting universe for a billion years, the first two green steps. This is shown repeated two more times. It can be seen that we are creating an equiangular or logarithmic spiral with a characteristic angle of 45 degrees, shown in orange. This diagram can be used to determine the co-moving radial distance, d_c, to any object with a given red shift. Take an object at point A in the empty, or 3-sphere, universe, when the universe was a half of its current size and a half of its current age. This object will have a cosmological red shift of 1. The object emits a photon which follows the orange logarithmic spiral to the observer, while the object itself travels with the expanding universe to the point R1 in today's universe. The distance 'O' to R1, around the universe is the co-moving radial distance, d_c, to any object with a red shift of 1. The formula for a logarithmic spiral[5] can be used to calculate that distance.

$$r = A \exp(\Theta \cot b) \qquad \text{Equation 7}$$

Since b, the characteristic angle, is 45 degrees, cot b is 1. r is the distance from the origin to the logarithmic spiral. A can be set to r_0, when Θ is the angle in radians, d_c/r_0. Dividing both sides by r_0 gives r/r_0, *a* or $1/(z+1)= \exp(d_c/r_0)$. But this is for a growing spiral. For a diminishing spiral the formula is $(z+1)= \exp(d_c/r_0)$, which is identical to the solution of the analytical equation, Equation 6. This is very strong additional evidence that the empty universe is in fact a 3-sphere expanding at the speed of light.

One interesting feature of Figure 1 is that, going backwards in time, the photon circles the centre, getting ever closer but never reaching it. The centre, in Figure 1, is not and has never been, part of the universe.

5. Is the empty universe really empty?

The last section could have finished with further discussion of red shift in the empty universe, but that is rather pointless if the empty universe is truly empty, there is nothing that can be red shifted. So the emptiness of the empty universe is discussed next.

If we assume the universe is a 3-sphere, everything within the 3-sphere exists at a distance r_0 from the centre. The mass of the shell has been discussed in Section 3, the mass equivalent of the curvature energy, while the same mass nominally resides at the centre of gravity. The force of gravity acting on the shell is

$F = M^2G/r^2$, Newtons   Equation 8

Substituting for M from Equation 5 gives

$F=c^4/4G$, Newtons   Equation 9

So if the universe expands by 1 metre, the additional gravitational potential energy is given by Equation 10. Note the 1m has been removed but for dimensional consistency it should be left in.

$\Delta E_{gpe} = c^4/4G$, Joules   Equation 10.

The mass of the universe is given by Equation 5 so the additional mass for a 1m expansion is, Equation 11

$\Delta M = c^2(r+1)/2G - c^2r/2G$ Equation 11

Multiplying by c^2 to give energy, gives Equation 12 (As for Equation 10, 1m should be left in)

$\Delta E = c^4/2G$ Equation 12

Exactly 50% of the total increase in energy as the empty universe expands is the increase in gravitational potential energy. This has been happening since the big bang, so exactly 50% of the total energy of the empty universe at all times is gravitational potential energy.

This immediately leads to a problem. There is no room for this potential energy in the empty or curvature-only universe, since all of its energy is ascribed to that of curvature. So how can this potential energy exist? There is a way around this problem. The potential energy calculated above is negative. If it was balanced by components with positive energy, then the total energy of the components, including the potential energy, would be zero, they would be invisible to the FLRW metric. How might this work? We could assume 30% of the energy, or mass, of the universe, was matter, 5% baryonic and 25% dark. This would be travelling with the universe at the speed of light so might have a kinetic energy of ½ Mc^2, (using Einstein's dispersion equation instead would, for a body travelling at the speed of light, give 0.414 Mc^2, not that different) 15% of the total. There would presumably be a trace of radiation, leaving 5% for vacuum energy. If there were no vacuum energy, the composition would not change significantly. With 30% matter and 70% energy, in this respect at least, the so called empty universe is similar to the standard model. But if the empty universe can contain components, is it appropriate to call it the empty universe? For the remainder of this paper it will be called the 3-sphere universe or model. With the possibility that the 'empty' universe need not be empty, it perhaps needs a more detailed investigation as to how it compares with other models of the universe.

The total mass or mass equivalent of the components is the same as the mass equivalent of the curvature but it is not additional mass. That would be double counting. They are one and the same mass.

To begin with, a 3-sphere universe with components will experience red shift. It is now appropriate to continue the discussion of red shift that was left over from the previous section. Figure 2 shows the scale factor vs distance relationship for the 3-sphere model and, for comparison, that for the lambda-CDM model, taken from Ned Wright's Cosmology Calculator[6].

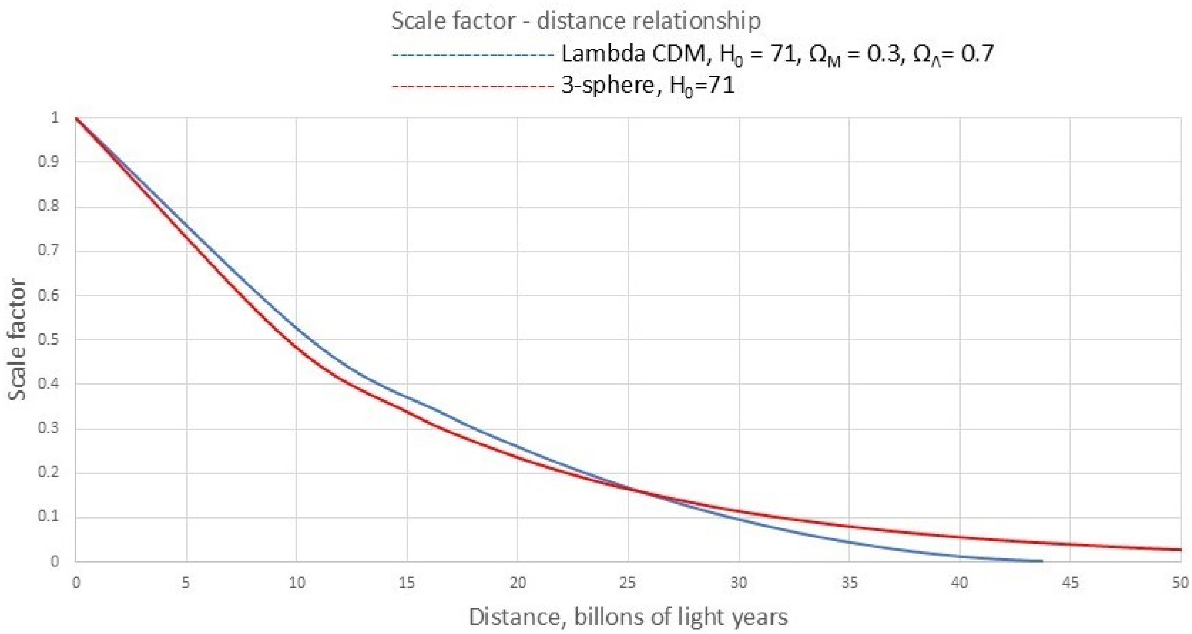

Figure 2. Scale factor vs distance

The relationship is very similar in the two models, certainly up to double digit red shift values, to the extent that supernova data from Perlmutter et al is unable to distinguish between the two models. Nb the parameters of the lambda-CDM model have been adjusted to ensure a good fit to the data. The 3-sphere model has no parameters to adjust. What you see I what you get.

Now that it seems the 'empty' universe need not be empty, it perhaps deserves more consideration as a possible model of the universe. In this respect it is interesting to look at the ages of objects at very high red shift values (low scale factors). These are shown in Figure 3.

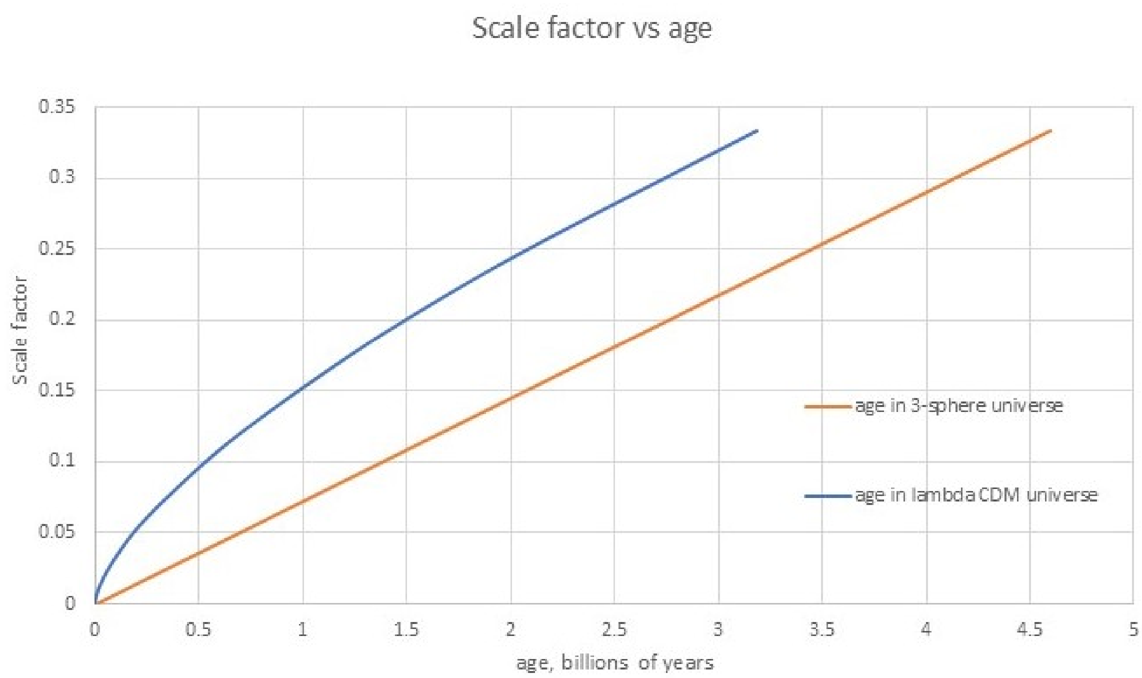

Figure 3. Scale factor vs age for the 3-sphere and lambda CDM models, parameters as for Figure 2.

What can be seen is, that at these low scale factors, in the very early universe, objects in the 3-sphere model are considerably older than objects in the lambda CDM model, around 4, 3 and 2 times for red shift values in the region of 20, 10 and 6 respectively. This would be highly advantageous to modellers of early stars, galaxy formation and structure in the universe where the lambda CDM model leaves barely, if at all, enough time for these things to happen.

6. Winding back the 3-sphere universe

It is informative to wind back the 3-sphere universe towards the big bang. This is shown, qualitatively, in Figure 4. The radius and mass decrease linearly to zero, the volume is proportional to the radius cubed, while entropy is proportional to the radius squared. The density however is inversely proportional to the radius squared and will rise to infinity at zero radius. The black hole temperature is inversely proportional to M, or r in the 3-sphere universe, and will also rise to infinity at zero radius. The singularity, at time zero, is all property and no substance and could be called a Cheshire Cat singularity.

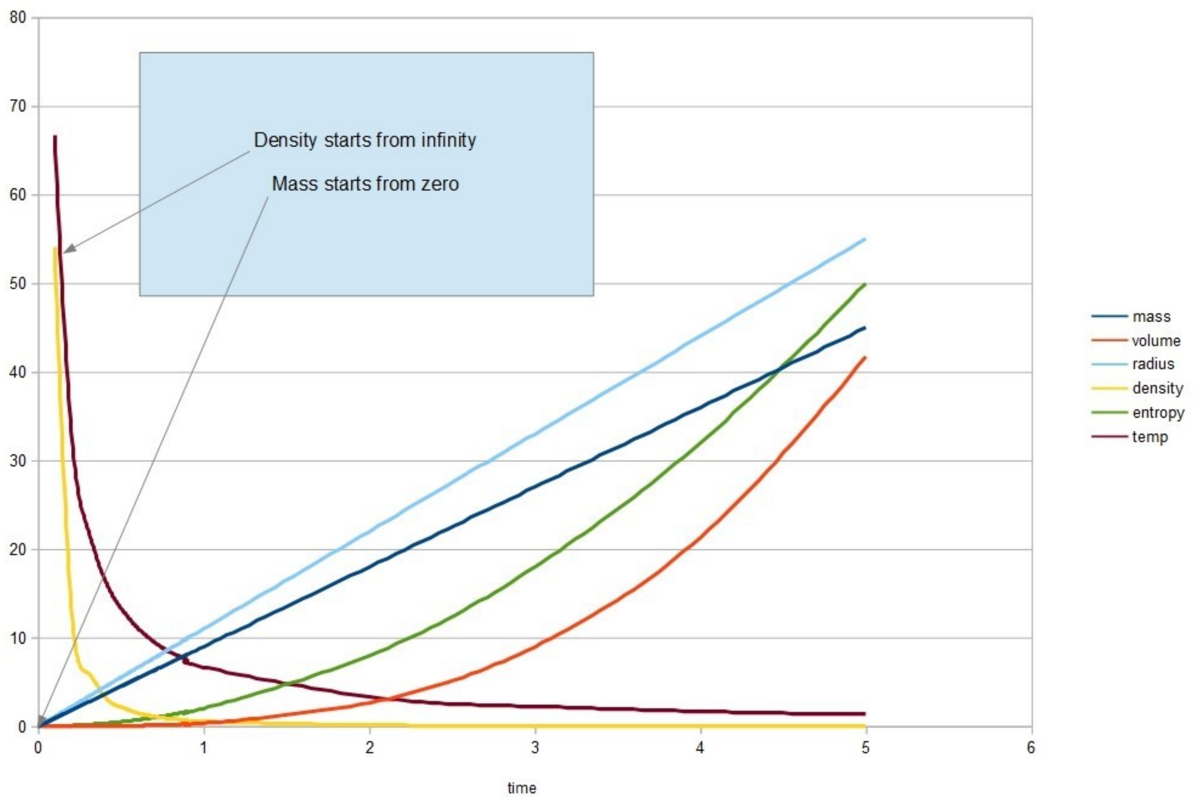

Figure 4. Winding back the 3-sphere universe.

Of course it is likely that quantum physics will intervene before the universe can shrink to zero size. The Planck scale is one candidate for the smallest size of the universe. When the 3-sphere universe is one Planck time old it will have a radius of one Planck length. Its mass will be one Planck mass, the same as its black hole mass. But it will have a truly enormous density, in the region of 10^{95} kg/m^3. Its temperature will be similarly enormous, with a black hole temperature in the region of 10^{31} K or the Planck temperature of 10^{32} K. The density and temperature at the Planck scale are not of course infinite, but they are truly enormous.

7. A remarkable calculation.

All components of the universe have an energy and therefore an energy density. Energy density has the same units as pressure and pressure is an input into the stress, energy, momentum tensor, one of the two main tensors to form Einstein's theory of general relativity. But energy density does not equal pressure, it is related to pressure by the equation of state, Equation 13.

$$p = \omega \rho \qquad \text{Equation 13}$$

p = pressure
ω = a dimensionless constant
ρ = energy density

In the standard model at least, each component has its own value of ω. That for matter is zero, it gives rise to no pressure, that for dark energy is -1, the minus sign indicating that the resultant force opposes that of gravity. It is possible to calculate ω using equation 14[7].

$$\omega = n/3 - 1 \qquad \text{Equation 14}$$

n is the exponent of (1-z) in Equation 1. n for curvature is 2, so ω = -1/3. In fact all coasting universes, that is all universes that are expanding (or contracting) at a constant rate, must have an ω value of -1/3. Might there be a more transparent way of calculating ω?

If the universe is coasting, then, according to Newton's 1st Law, there must be zero net force acting upon it. But we know that in a 3-sphere universe there is a force of gravity as given by Equation 8. So the force due to the pressure, as given by equation 13 must exactly balance the force due to gravity.

$$F_{gravity} + F_{pressure} = 0 \qquad \text{Equation 15}$$

But what is $F_{pressure}$? Normally the pressure is entered into the stress energy momentum tensor and never seen again. Force equals pressure times area. If we had an appropriate area then the force would be simply ω x ρ x area. The 3-sphere does, however, have an area, the derivative of its volume, equal to $6\pi^2 r^2$. This is not a surface area since the 3-sphere is itself a surface, but it is an area nonetheless. It is numerically equal to the increase in volume when the universe expands by 1 metre which perhaps makes it seem more concrete. In today's universe, for every cubic metre of space, there is roughly 10^{-26} m^2 of area, we wouldn't notice it, but in the Planck sized universe there would be roughly 10^{-33} m^3 of volume for every square metre of area. It might be fair to say that the Planck sized universe is more 2-dimensional than 3-dimensional.

By setting the energy density of curvature in the empty or 3-sphere universe to the critical density and combining Equations 8, 13 and 15, we are ready for the remarkable calculation.

$$F_{gravity} = M_{universe}^2 G/r_0^2 \qquad \text{Equation 16}$$

Substituting for $M_{universe}$ from Equation 4 and cancelling, gives

$$F_{gravity} = c^4/G \qquad \text{Equation 17}$$

Again, taking the energy of the universe from Equation 4

$$\text{Pressure} = \omega\, c^4 r_0/ (G\, 2\pi^2 r_0^3) \qquad \text{Equation 18}$$

So the force equals

$$F_{pressure} = \omega\, c^4 r_0/(G\, 2\pi^2 r_0^3) \times 6\pi^2 r_0^2 \qquad \text{Equation 19}$$

Substituting into Equation 15

$$c^4/G + \omega\, c^4 r_0/(G\, 2\pi^2 r_0^3) \times 6\pi^2 r_0^2 = 0 \qquad \text{Equation 20}$$

Rearranging and cancelling

$$\omega = -1/3 \qquad \text{Equation 21}$$

The same as Equation 14 when n = 2. By ascribing an area to the 3-sphere universe, a very simple calculation gives the correct value of ω to the coasting 3-sphere universe. The same result is obtained if Equation 5 is used in place of Equation 4. In fact the result is simply related to the exponent '3' in all units of volume, when it is used in calculating the derivative, area.

8. Time dilation as in special relativity

The empty or 3-sphere universe is expanding at the speed of light, in the direction of time. A very simple application of Pythagoras' theorem calculates time dilation.

In Figure 5, the axes are in light seconds, the x-axis is space and the y-axis is time and there is an observer at the origin. The diagram covers 1 second. If there is no movement through space, objects move with the expanding universe to cover a distance of 1 light second, the yellow line. But if there is movement through space, relative to the observer, this is shown by the red line – in this case the velocity is 0.5 c, so the distance covered in 1 second is 0.5 light seconds. By vector addition, the distance travelled in 1 second is given by the orange line, which is faster than the speed of light, which is obviously not allowed. The distance travelled through time must reduce, to x, until the length of the blue line is equal to 1 light second, giving Equation 22.

$$\sqrt{(x^2 + v^2)} = 1 \qquad \text{Equation 22}$$

Or

$$x = \sqrt{(1-v^2)} \qquad \text{Equation 23}$$

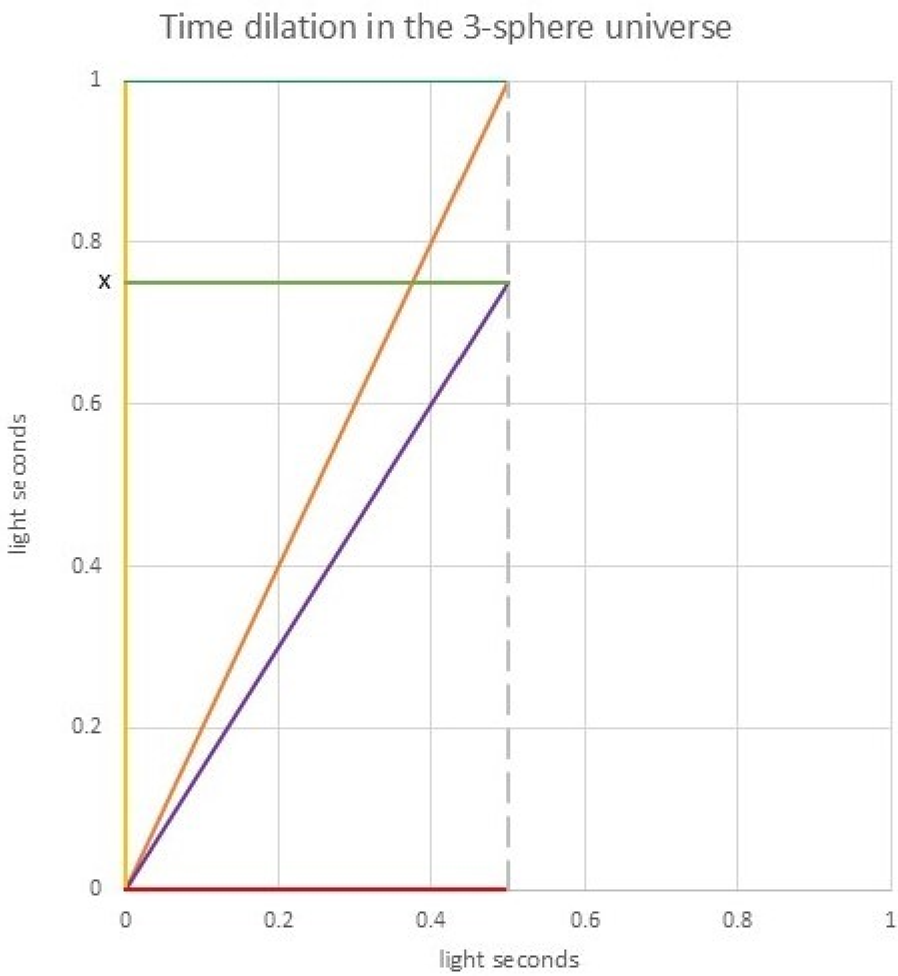

Figure 5 Time dilation in the empty or 3-sphere universe

x could be viewed as a factor by which the speed of light slows down, but the constancy of the speed of light is a cornerstone of modern science. Instead, x is viewed as the rate by which time slows down, time dilation. Time dilation is usually viewed as the slowing of the rate of ticking of a clock. The time period between clock ticks gets longer. This leads to the more common formula.

$$\Delta t'/\Delta t = 1/\sqrt{(1-v^2)} \qquad \text{Equation 24}$$

$1/\sqrt{(1-v^2)}$ is the Lorenz γ factor (simplified when distances are measured in light seconds) that appears extensively in special relativity

9. Evidence for a 3-sphere universe

So far, it has been shown that the empty universe need not be empty and that the resulting 3-sphere model has some interesting properties. Now it is time to try some more meaningful comparisons between the 3-sphere and lambda CDM models. This is done in this and the next section.

Here the number of objects seen on looking deeper into space is compared between the two models and with data. The assumption is that the number of objects observed is proportional to the volume of space observed, which for the lambda CDM model, with its non-curved space, is simply proportional to the cube of the distance observed. The incremental number observed will be proportional to the square of the distance observed. There is an additional factor, though, for the 3-sphere universe. The lambda CDM model has a constant mass, at least of matter. In the 3-sphere universe, on the other hand, the total mass is proportional to the radius. It is reasonable, as a first approximation at least, to expect that the total number of objects will also be proportional to the total mass and hence the radius of the universe.

The other issue for the 3-sphere universe is that the author was unable to find a distance volume relationship for a 3-sphere so it has been calculated from first principles. The prompt for the calculation was the observation that the volume of a 3-sphere of radius r is equal to the area of a circle of radius r times the circumference of a circle of radius r. Perhaps the volume observed by looking for a distance through a 3-sphere could be found by integrating the areas of circles with respect to the distance moved around the circumference of a circle as shown in Figure 6. In this diagram the main circle is represented by the red spokes. Each circumferential circle is centred on a spoke and has a diameter equal to the corresponding chord of the main circle. But this scheme produced a total volume for the 3-sphere one quarter of the correct value. This could be corrected by assuming that each circle in Figure 6 was in fact a 2-sphere with an area equal to $4\pi r^2$, four times that of a circle. In three dimensions these 2-spheres are all overlapping. It has to be assumed that in four dimensions, this is not the case. The resulting integral is

$$\text{Volume} = 4\pi R^2 \int_0^{\pi R} \sin^2(r/R)\, dr \qquad \text{Equation 25}$$

This is a standard integral[8] with the solution

$$\text{Volume} = 4\pi R^2 \left[r/2 - \frac{\sin(r/R)\cos(r/R)\, R}{2} \right]_{r=0}^{r=\pi R} \qquad \text{Equation 26}$$

R is the radius of the 3-sphere and r is the distance looked around the 3-sphere, r/R being an angle in radians.

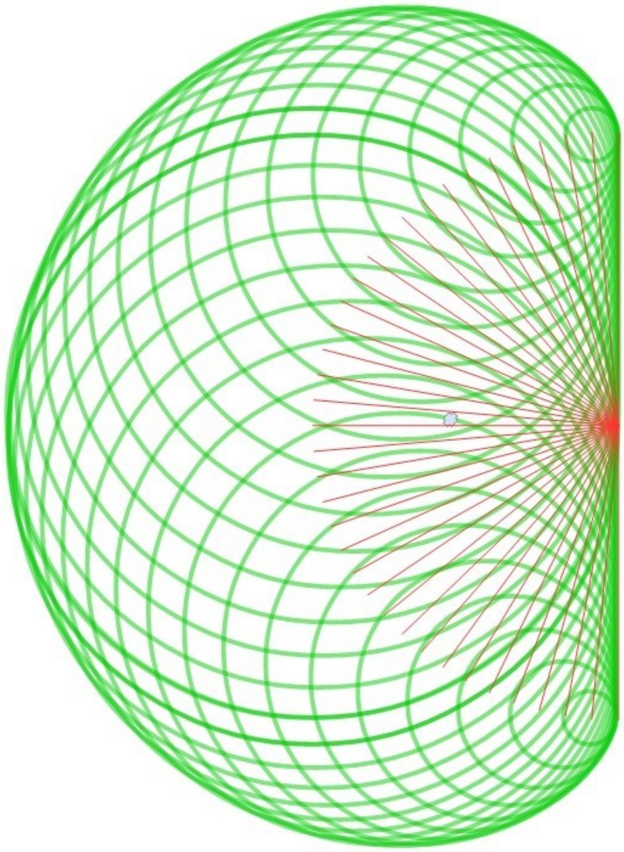

Figure 6 2-spheres centred on the circumference of a circle, a construction used to calculate the volume in view on looking around a 3-sphere.

Just for interest, if each 2-sphere is slid along the radius of the circle, for a distance equal to the radius of the 2-sphere, Figure 7 results. This is still only a construction used in a calculation but it is a possible 2-dimensional representation of a 3-sphere.

To recap; it is postulated that the number of objects seen on looking deeper into space will be proportional to both the volume of space seen and the total mass of matter in the universe at that time. The latter is of course constant for the lambda CDM model but not for the 3-sphere model. The result is shown in Figure 8. Red shift is used instead of distance in order to test against data from the Sloane Digital Sky Survey, SDSS. There is only data on galaxies up to red shifts of around 2.5, which is not sufficiently high for this test, so the data on quasars is used which are listed up to red shifts of around 7. The red shift distance relationship, for the lambda CDM model, is taken from Ned Wright's cosmology calculator, while that for the 3-sphere model is taken from (z+1) = $\exp(d_c/r_0)$, (para following Equation 7). Neither derivation uses the actual density of quasars in the universe, each curve is simply scaled to match the SDSS data as well as it can, primarily by matching their maximum points.

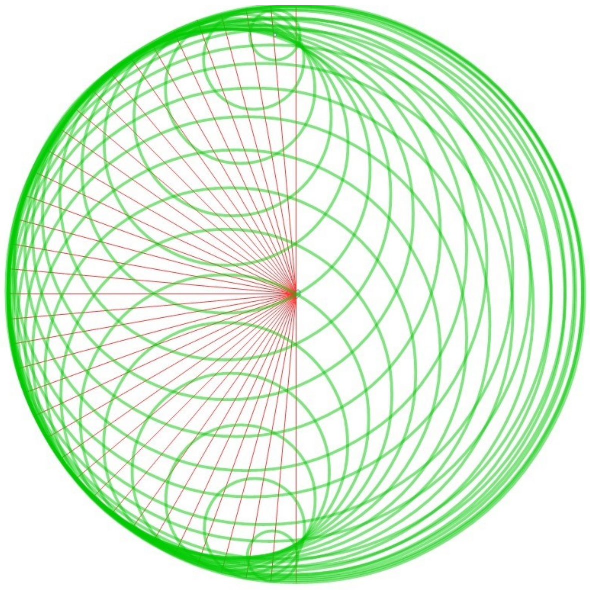

Figure 7 A possible representation of a 3-sphere

There will be other factors at play, such as obscuration by gas and dust clouds, but even after allowing for this, the prediction of the 3-sphere model is far superior to that of the lambda CDM model.

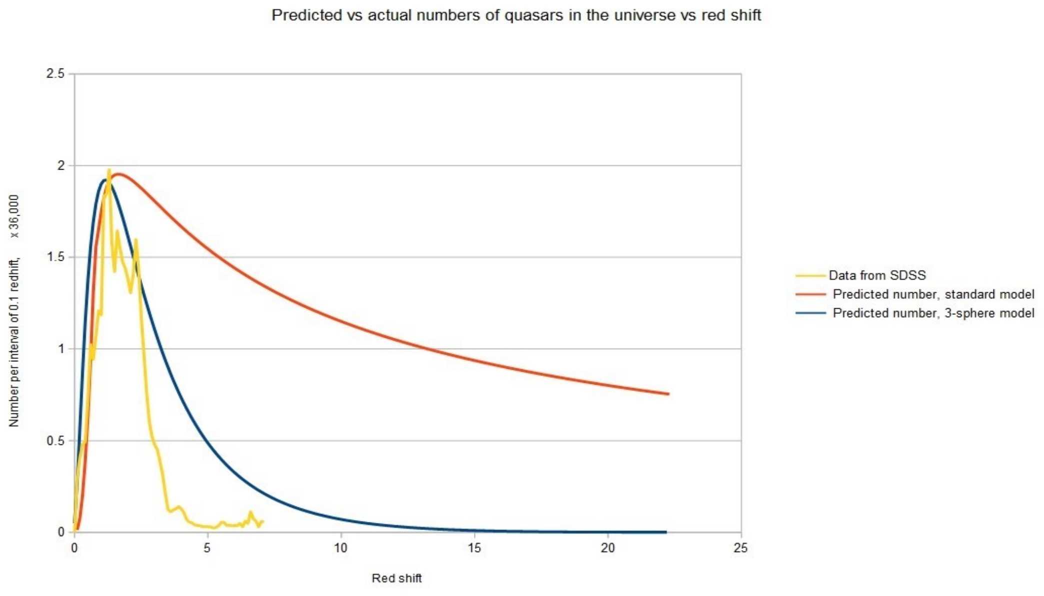

Figure 8 Predicted numbers of quasars in view compared against SDSS data

10. Time dilation in a 3-sphere universe

Once again, the same result is calculated in two very different ways, which always seems to add weight to the conclusion. The first starts from a development of Figure 1. And is shown in Figure 9.

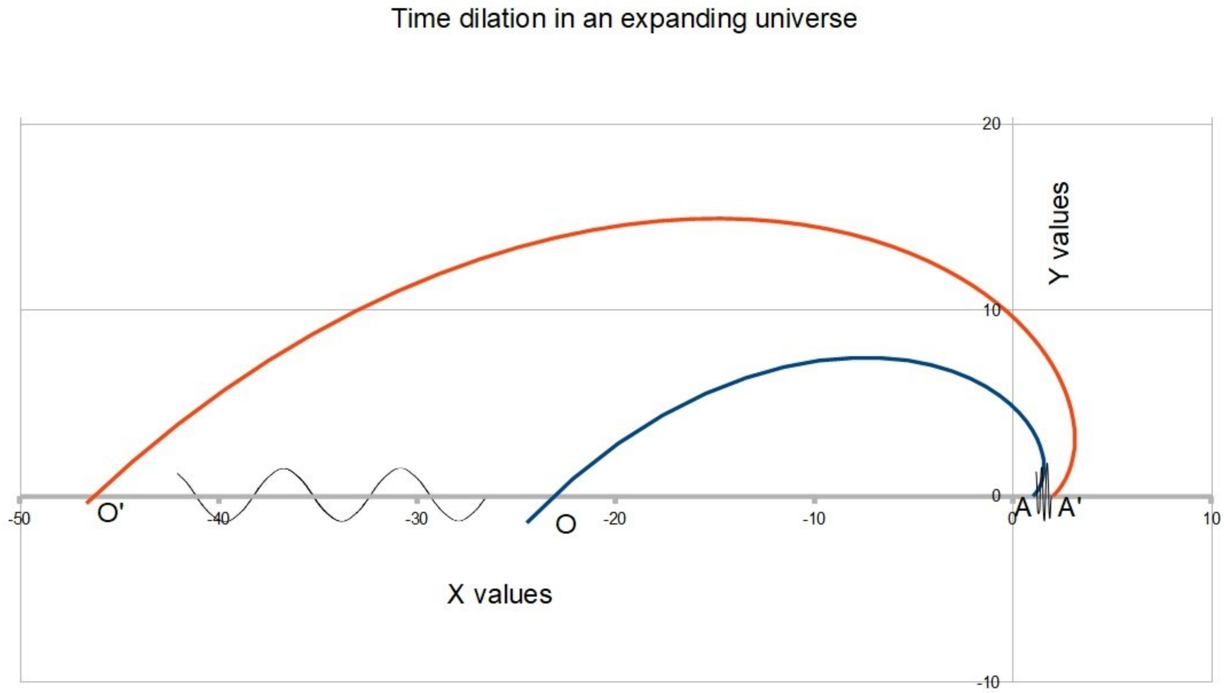

Figure 9 Time dilation in an expanding 3-sphere universe

A is a galaxy in the 1-billion-year-old universe. A emits a photon which follows a logarithmic spiral, the blue curve, as in Figure 1, until it meets an observer O, at A's antipodean point, in the 23-billion-year-old universe. The scale factor of A, relative to O, is 1/23. The galaxy then moves with the expanding universe to A' in the 2-billion-year-old universe and emits a second photon which also follows a logarithmic spiral, the orange curve. The observer at O is also moving with the expanding universe and encounters the second photon at O' in the 46-billion-year-old universe. The scale factor is still 1/23. But both the observer and the galaxy are moving with the universe at the speed of light. This is only possible if we have time dilation. The observer will see a clock in the galaxy running at 1/23 the rate of their own clock. The wavelength of a signal in the galaxy will be seen as stretched, as shown, by this same factor, 23. Light will be seen as red-shifted by a factor of 22.

At first sight, this might seem to lead to a difficulty. Cosmological red-shift in the expanding 3-sphere universe is 100% explained by time dilation. There is no room for any Doppler shift. But reflection shows that this seems to be correct. The first motion of the photons leaving the galaxy A, is vertical, parallel to the Y-axis. But the motion of the galaxy, along with the expanding universe, is along the X-axis, perpendicular to the motions of the photons. In fact the motion of the photons is at all times perpendicular to the direction of expansion. Motion perpendicular to the direction of a travelling signal does not lead to any Doppler shift. An interesting corollary here is that, if the universe has curvature, there will be no cosmological Doppler effect.

The paths of the photons, the blue and orange curves, are geodesics. There will be a force of gravity in the vicinity of the curves that is a function of their curvature. It can be seen that the nearer each curve gets to the origin, the smaller will be its radius of curvature, the greater will be the force of gravity. It looks as if the time dilation in Figure 9 is gravitational time dilation. The remainder of this section will show that this is correct.

Gravitational time dilation can also be estimated analytically, using Equation 27 [9].

$$T_d(h) = \exp[1/c^2 \int_0^h g(h)\, dh] \qquad \text{Equation 27}$$

$T_d(h)$ = the time dilation factor at height h.
g(h) = the force due to gravity at height h.

For a spherical universe this can be re-written as Equation 28.

$$T_d(r) = \exp[1/c^2 \int_{r_1}^{r_2} g(r)\, dr] \qquad \text{Equation 26}$$

$T_d(r)$ = the time dilation factor for a universe with radius r.
g(r) = the force due to gravity from the rest of the universe for a universe of radius r.

Sticking with this notation, the gravitational force on a unit mass in a 3-sphere universe of mass M, is MG/r^2. Substituting for M from Equation 4 results in Equation 27

$$g = c^2/r \qquad \text{Equation 27}$$

The integral of this is $g = c^2 \ln(r)$. Putting this into Equation 26, cancelling c^2 and solving, results in Equation 28

$$T_d = \exp[\ln(r_2) – \ln(r_1)] \qquad \text{Equation 28}$$

Or

$$T_d = r_2/r_1 \qquad \text{Equation 29}$$

The time dilation factor between the 3-sphere universe at any two ages is simply the ratio of the radii or scale factors. This is exactly what Figure 9 shows. In Equation 29, r_1 will generally be r_0 the radius of today's universe. This confirms that in the 3-sphere universe there is no room for any cosmological Doppler effect, probably true for any universe with curvature.

While this section has made no comparisons with the lambda CDM model, its massive internal consistency would seem to lend significant weight to considering the 3-sphere model as a contender model for the universe.

11. Counter arguments

Of four counter arguments to the idea that our universe might actually be an expanding 3-sphere, one has already been addressed, the second is discussed here, the third is left to the next section and the fourth is left to an admittedly highly speculative final section.

The SN1a red-shift distance relationship established by Perlmutter et al has been claimed to demonstrate that the expansion of the universe is accelerating. But this is only correct if it is first assumed the universe is flat. In fact, as Section 5 shows, this data is fully compatible with a 3-sphere universe expanding at a constant rate. For single figure values of red-shift this relationship is very similar in the lambda CDM and 3-sphere models. It is suggested though that the more leisurely pace of the red-shift age relationship of the 3-sphere model should be welcomed by many researchers into the early universe.

Data from the analysis of the power spectrum of the fluctuations in the cosmic microwave background, CMB, from the Boomerang and subsequent measurements, are said to show that our universe must be flat. This is because the ratio between the size these fluctuations are thought to have had, and that observed in today's CMB, is equal to the scale factor, an indication that the universe is indeed flat. This is based on the idea that the age of the universe at the period of recombination is around 375,000 years, making the size of the fluctuations around 375,000 light years. But in the 3-sphere model the age of the period of recombination is closer to 10 million years, making the original fluctuations much larger. If the Boomerang calculations are repeated it must be concluded that we are seeing these fluctuations to be much smaller than they really were, due to the curvature of the universe. Note that the red shift and scale factor of recombination is a function of the temperature of the universe at this time and this is the same in all models, a function of the ionisation potential of hydrogen atoms.

If the Boomerang calculations start by assuming an early time for recombination, ie that the universe is flat, they conclude that the universe is flat but if they assume a later time for recombination, that the universe is curved, they conclude that the universe is curved. This is not proof of anything.

The third counter argument centres on whether the curvature of the universe is positive or negative. This is addressed in the next section.

The fourth counter argument concerns the synthesis of the elements, primarily helium, in the very early universe. The Lambda CDM model allows for a compelling story, where hydrogen fuses to form helium in the first tens of minutes following the big bang. In fact the 3-sphere universe passes through conditions very similar to those of the early Lambda CDM universe and would generate helium by the same route, but at this time the 3-sphere universe would have weighed only a tiny fraction of its present-day mass; most of the helium observed could not have been generated at this time. No solutions to this problem are proposed, save for some highly speculative ideas in the final section.

12. The elephant in the room

The elephant in the room is this. Much of this paper is centred around the idea that the empty universe is a 3-sphere expanding at the speed of light. A 3-sphere, just like a 1-sphere, a circle, or a 2-sphere, a bubble or balloon skin, has positive curvature. But in the empty universe the relative energy density of curvature is +1. This corresponds to negative curvature. The empty universe appears to be hyperbolic. A possible way out of this conflict is shown in Figure 10.

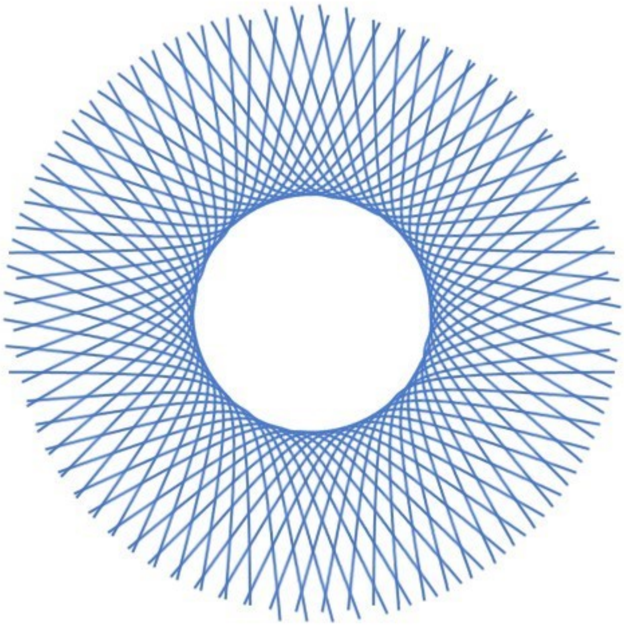

Figure 10 Shape and curvature

In Figure 10, the central white space can be viewed as a 2-dimensional disc. It has an outside boundary, a 1-sphere or circle, with positive curvature. Each of the blue lines is a 1-dimensional hyperbolic curve. If a single curve is rotated by 360 degrees about the centre, it forms a 2-dimensional hyperbolic space as shown. But this 2-dimensional hyperbolic space has an inner boundary identical to that of the disc, a 1-sphere or circle with positive curvature. This appears to resolve the conflict. 4-dimensional spacetime can have positive or negative curvature, but both will have 3-sphere surfaces with positive curvature.

13. Some speculation

The main conclusion of this paper, that the FLRW empty or Milne universe, need not be empty and, as such, deserves serious consideration as a model for the universe, may seem rather speculative. This section, however, takes speculation much further but if it is wrong it should not detract from the rest of the paper. The first two or three speculations may appear tangential to this paper so far, but if they are correct, they would lend support to the later speculations.

a. black holes

An expanding 3-sphere universe has a number of properties of a black hole, most notably a mass which is directly proportional to its radius. This leads naturally to the following proposition. If black holes are 4-dimensional balls of spacetime, why won't they have 3-sphere 3-dimensional surfaces. Then, all of the matter and radiation that enters a black hole would remain in the 3-sphere surface, situated at the event horizon. This would avoid a number of paradoxes associated with black holes.

1 There would be no physical singularity; the singularity would be no more than the centre of gravity. All paradoxes associated with such a singularity, even as modified by quantum physis, disappear.

2 Time slows to zero as the force of gravity rises to infinity. This leads to the conclusion that time dilation should be infinite at the singularity. But, in the standard view, this property of the singularity, along with others of its properties, gets transferred to the event horizon. This is easily explained if time, for a black hole, is a function of motion towards the singularity. In a 3-sphere black hole, nothing moves towards the singularity from the event horizon, so time must stop there.

3 In the standard view of black holes it is commonly understood that as matter passes through the event horizon, it is mysteriously separated from its entropy, which remains at the surface as the area of the black hole. In a 3-sphere black hole, matter remains in the 3-sphere surface and need never get separated from its entropy.

b. A minimum size for black holes

Imagine a 3-sphere black hole that consists solely of radiation. Let this shrink due to Hawking radiation. When it reaches a diameter equal to the Planck length, it will have the Planck mass. A photon with a wavelength equal to 2 pi times the Planck length, also has the Planck mass. A 3-sphere black hole can't contain a photon less massive than this, because its wavelength would be greater than the circumference of the black hole but it can't contain a photon with a wavelength shorter than this because then it would weigh more than the black hole. Such a 3-sphere black hole can't shrink any further because as a black hole it would have to get less massive, but as a photon it would have to get more massive. Since it can't do both at the same time, it can't shrink any further. Such a 3-sphere black hole, which can't shrink any further, then becomes an extremely good candidate for dark matter.

c. The evolution of the 3-sphere universe

For the purposes of this thought experiment it can be imagined that the universe is ticking, with a tick equal to the Planck time. At the end of the first tick, a 3-sphere universe, expanding at the speed of light, will have a radius equal to the Planck length and a mass equal to the Planck mass (it will have the enormous temperature and pressure described in Section 6). It will be, simultaneously, a 3-sphere, a black hole and a single photon with a wavelength equal to the circumference of the 3-sphere.

At the end of the second tick, the expanding universe will have double the radius and therefore double the mass, two Planck masses. But the wavelength of the circumferential photon will have doubled, giving it one half of the Planck mass. Four such photons are now required. After three Planck ticks there will be nine photons, each with one third the Planck mass, giving the universe a mass of three Planck masses, and so on. Fast forward to the present day. The universe is roughly 10^{61} ticks old. In the next tick, 2 x 10^{61} photons are generated, each with a wavelength of roughly 88 billion light years, the current circumference of the 3-sphere universe. This is closely equal to the Planck mass. The total mass of the universe is equivalent to 10^{122} of these photons, a mass equivalent of 10^{53} kg and a density equal to the critical density.

Of course, today's universe does not consist solely of photons, so to continue the speculation. The one Planck length universe has an area equal to $6 \pi^2 r^2$, roughly 60 square Planck lengths (actually 6

π^2). After two ticks, the area will have increased by a factor of four, and the number of photons has increased to four. Similarly after three ticks, both the area and the number of photons will have increased nine-fold. At all times, under this scheme, the area per photon is a constant at 60 square Planck lengths. Perhaps this actually is the area of a photon or, more likely, the area of universe available to each photon. Continuing the speculation, it is extremely tempting to multiply this area by the wavelength to obtain a volume associated with each photon. When this is done, the perhaps surprising result is obtained that the total volume of photons generated by this scheme is always twenty ($2\pi^2$) times that of the 3-sphere volume of the universe.

This state of affairs doesn't seem possible. 95% of these photons will be forced to condense in order to lose volume, most probably into a mixture of baryons and single Planck length black holes, dark matter. Allowing for the fact that these components will have associated potential and kinetic energies will give a composition along the lines proposed in Section 5.

This scheme suggests the universe may contain around 5% background or vacuum energy consisting of universe sized photons. Our chances of interacting with photons of such a massive wavelength may seem minimal but the fact that they are supersaturated and continually condensing may allow for some possible interaction.

Conclusion

The main conclusion is that the FLRW empty universe, need not, in fact, be empty. The resulting model, a 3-sphere, containing components such as matter, and expanding at the speed of light, has very interesting properties and is proposed as a contender model for the universe.

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References

[1] Sean Carroll, Sean M. Carroll, 2004, Spacetime and Geometry: An Introduction to General Relativity, p341. Addison Wesley, ISBN 9780805387322.

[2] M. Pettini: Introduction to Cosmology — Lecture 5. Redshifts and distances in cosmology. University of Cambridge. Lecture05.pdf.

[3] Andrew Liddle, 2003, An Introduction to Modern Cosmology, (2nd Ed), Wiley–Blackwell, ISBN-10. 0470848359

[4] David W. Hogg, 2000, Distance measures in cosmology, Institute for Advanced Study, 1 Einstein Drive, Princeton NJ 08540. arXiv:astro-ph/9905116v4

[5] Eric W. Weisstein, "Logarithmic Spiral." From MathWorld-A, Wolfram Web Resource. https://mathworld.wolfram.com/LogarithmicSpiral.html

[6] Ned Wright's Cosmology Calculator Wright (2006, PASP, 118, 1711).

[7] Fulvio Melia, 2023, The Friedmann–Lemaître–Robertson–Walker metric and the principle of equivalence, Zeitschrift fur Naturforschung A, Volume 78 Issue 6

[8] https://www.math.stonybrook.edu/~bishop/classes/math126.F20/CRC_integrals.pdf

[9] John B. Kogut, 2012. Introduction to Relativity: For Physicists and Astronomers (illustrated ed.), p.112, Academic Press. ISBN 978-0-08-092408-3.